FRACTIONS ON THE NUMBER LINE: THE TRAVEL OF IDEAS

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This study analyzes the emergence and travel of ideas about fractions on the number line in three upper elementary school mathematics classrooms. The findings indicate that students’ public displays of ideas shifted during the course of the lesson, classrooms varied in their treatment of the number line, and shared discourse within classrooms was rooted in different understandings.

A principal canon of inquiry-oriented mathematics classrooms is that teachers use student thinking as an instructional resource. In their instructional moves, teachers pose problems, solicit contributions, and orchestrate discussions in ways that lead students to make public their ideas and put their ideas in relation to those of others. In this process, students’ mathematical ideas emerge and become elaborated, sometimes refined, and often transformed in classroom communities. Over the course of such classroom discussions, ideas may be taken up or rejected, valued or devalued, and understood differently by individual students in the community. The travel of ideas may take varied trajectories, and these trajectories may vary across classrooms implementing the same lesson. Thus, not only may the ideas that are taken up by students differ across classrooms, but also students within a given classroom may understand the same ideas in varied ways. Understanding the emergence and travel of ideas -- what travels and how -- is basic to understanding learning and teaching in inquiry-oriented classroom communities. It also requires new ways of extending and elaborating empirical and conceptual frameworks.

The purpose of this paper is to present an analysis of the travel of mathematical ideas in three upper elementary school classrooms, each implementing the same lesson sequence involving fractions and number lines. In the paper, we focus on a single lesson across the three classrooms. The lesson presented a non-routine Problem of the Day (PoD) that involved the identification of a fractional point on a number line. In the problem, the number line was partitioned into unequal intervals, and the ‘big idea’ of the lesson was to support students in understanding the function of partitioning the line into equal intervals in conceptualizing the fractional value of a point. Our focus on the travel of ideas is important, particularly for understanding emergent mathematical environments and learning opportunities in such inquiry-oriented classrooms.

Though existing empirical approaches to the study of teaching and learning have made important contributions, they reveal in only very partial ways processes whereby ideas travel. For example, quantitative efforts have made use of rating scales to capture the depth of coverage of subject matter and the extent to which teaching builds on assessment of student thinking (e.g., Gearhart et al., 1999; Saxe, et al.,1999). Though these studies point to general processes that enhance learning in inquiry-oriented classrooms, they do not reveal well processes whereby ideas travel and become elaborated or put aside. Qualitative analyses describe with greater texture the interplay between learning and teaching in classrooms.
including teachers’ instructional decisions (Ball, 1993) and representational contexts (Cobb, 2002; Lampert, 2001; Sfard, 2002). But these studies typically do not follow the emergence and transformation of mathematical ideas in classroom life over a lesson, series of lessons, or even longer durations of time.

The Focal Lesson

The multi-phase lesson structure is depicted in Figure 1. It consisted of initial independent work on the problem, whole class and small group discussions, whole class problem resolution, and ended with independent work on problems that were similar to the PoD.

![Figure 1. Lesson structure for the PoD: Identifying a point on the number line](image)

**Methods**

Three teachers implemented the hour-long lesson in their untracked mathematics classrooms (1 fifth grade, 2 sixth grade) in an urban school district. We videotaped whole class and small group discussions, and interviewed a subset of students after class about the character of change in their thinking from their initial independent work to their current thinking. We also queried students both via written responses and in interviews about ways in which peers and the teacher may have influenced shifts in their thinking about the problem. In addition, we made use of a sociogram to identify students that were seen as particularly competent in mathematics, perceptions that we suspected might influence uptake and travel for some students.

**Results/Discussion Points**

Drawing upon our framework for studying the travel of mathematical ideas within classrooms (Saxe, Earnest, & Shaughnessy, 2007; Saxe, Shaughnessy, Earnest, & Cremer, 2007), we triangulated our data sources. In this process, we first coded worksheets to determine shifts in students’ approaches to solving the PoD, and we followed up these analyses with an examination of (a) case studies of uptake of ideas in whole class and small group discussion, (b) the relation of uptake to issues of social position (based upon sociogram data and self reports influence). The following are a subset of our findings and discussion points:

1. **Initial Student Ideas:** As expected, at the beginning of class, some students viewed the PoD as an incompletely partitioned number line, and added or removed hash marks to create intervals of equal length. But others made sense of the number line with fraction names like “2/6,” “2/7,” “2/4” and “2,” responses that suggested whole

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number reasoning (counting marks) and idiosyncratic understandings of number line conventions (confusions of the role of the zero, hash marks, or arrows).

2. Travel of Ideas: Students’ public displays of ideas shifted during the course of whole class and small group discussion. Students’ approaches to naming points on the number line were taken up, resisted, elaborated and coordinated in different ways in each of the three classrooms. Additionally, students varied in their approaches to treatment of 2/8 as a potential answer. Some students developed geometric arguments (removing hash marks) and other students developed arithmetic arguments (reducing) to argue that 2/8 = ¼.

3. Social Influences on Ideas: Analyses of student reports about who influenced shifts in their thinking and sociograms that revealed beliefs about their peers’ mathematical competence provide a basis to identify social sources of change – who, if any one, influenced change in thinking and why.

4. Comparative Analyses: In all classrooms, the number line was a focus of discussion; however, students’ use of the number line to solve the PoD varied by classroom.

5. Shared discourse and individual understanding: In all classrooms, students and teachers used collective language about fractions, points, and hash marks. But interviews with students revealed that what was shared in ostensibly coherent back-and-forth dialogue was rooted in different understandings.

References


